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A Set of Methods in Transportation Network Synthesis and Analysis

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This paper deals with the problem of modifying a transportation network so as to fit it with existing demand. A set of procedures were applied to the replanning of medium-sized town bus routes in several case-studies; the chosen approach, which is the result of five years research applied in about 10 towns in France, is described and results are presented and discussed for two cases.

INTRODUCTION

PERIODICAL changes in transportation networks are a necessity because of the quick evolution of the features of towns nowadays. Collective transport systems have to adapt themselves to real demand in providing a sufficient level of service, to avoid them being replaced by private cars. The choice of good alterations is a very difficult problem which cannot be solved simply because:

—the number of decision variables, as soon as the network is a realistic one, is rather high,
—interaction between service and usage is not numerically known,
—goals (i.e. what is a good network) are not always defined, and when they are, to express them mathematically is rather tricky.

Practically, the planner is compelled to define "by hand" various modifications he thinks are the best, and to test the resulting networks by means of simulation models. What is needed in fact, are procedures for network generation which could provide a small number of distinct alternatives, approximately estimated, to help the planner in his task—which is to prevent his forgetting solutions. Three subproblems were examined:

(A) To choose a set of streets.
(B) To choose a set of bus lines,
(C) To determine optimal frequencies.

Generation procedures deal with subproblems A and B. Computing frequencies can be achieved through a more precise model, taking into account the users' waiting times, but assuming both the street network and bus routes as fixed.

HYPOTHESES AND GOALS

The data

Nodes and links. The town which is to be provided with a network is divided into homogeneous zones: e.g. residential zones, working zones, etc. The size of a zone is such that every user can reach its centroid on foot within a reasonable walking time. Demand is assumed generated from the centroids. A set of nodes N is defined as the union of the set of centroids and that of main cross-roads.

$\overline{A}$ denotes the set of all main streets between nodes. $G = (N, \overline{A})$ is called the maximal network. Links in $\overline{A}$ are evaluated by the length of the streets, $d_\alpha \, \forall \alpha \in A$.

The demand. The demand for travel is known through sample surveys and with the help of trip distribution models (e.g. Wilson[1]). Each trip matrix is typical of an hour of the day, distinguishing work-home morning and evening trips. The rate of people using specifically collective vehicles is supposed to be given by modal choice functions which depend on the town but can change from a zone to another.

Other data. Apart from the maximal network $(N, \bar{A})$ and the trip matrix

$$D = [D_{ij}]\ (i, j) \in N^2,$$

consider also

—population and number in employment, P_i and E_i, in each zone i;
—a maximal cost allowed C_T for modifying the network structure;
—a maximal number of buses B_T which can be dispatched between the routes;
—the speeds V_α on each link α for the vehicles—these speeds depend on the usual traffic congestion in the streets and are determined through sample surveys;
—the cost of adding a street to the network, C_{ij}, $(i, j) \in \bar{A}$.

The unknown parameters are the set A of streets $(A \subset \bar{A})$, that of lines L and the frequencies of buses for each line. Each group of parameters must be evaluated separately, to avoid combinatorial explosion.

Goals. Generally, a good network is one with cheap investment and maintenance costs and with a sufficient frequentation rate. This latter point means that the transportation system is well-fitted to the actual demand. There will be two kinds of criteria: those for the cost, those for the level of service.

Costs. Costs will be represented by three terms:

$C = \sum_{i,j \in A} C_{ij}$ investment cost for using streets in A;

$\mathscr{L} = \sum_{\lambda \in L} l_\lambda$ total length of buslines λ belonging to the set;

$B = \sum_{\lambda \in L} B_\lambda$ total number of vehicles for the network; B_λ being the number of buses for line λ.

Level of service

We will have to minimize total travel time:

$T = \sum_{i,j \in N} D_{ij} t_{ij}$ for users.

t_{ij} is the travelling time between i and j, including waiting times for indirect trips. Waiting times are not included in subproblem A. Subproblem B attempts to minimize the number of waiting points.

Minimizing total travel time is not sufficient to take into account the feedback effect of the network upon the demand. The use of accessibility indices (Poulit[2]) allows the influence of the network upon the needs for travel to be measured without having to re-compute a trip matrix each time there is a network alternative.

Network can be chosen to maximize a global accessibility instead of a total travel time, especially in subproblem A.

FINDING AN OPTIMAL SUBSET OF STREETS

Formulation

The problem of finding an optimal subset of links in a maximal set is often called the optimal network problem (ONP). It is stated as:

$$\text{Minimize } T = \sum_{(i,j) \in N^2} D_{ij} t_{ij}(X)$$

$$\sum_{(i,j) \in \bar{A}} C_{ij} x_{ij} \leqslant C_T \quad \text{(investment constraint)}$$

$$x_{ij} = 0 \quad \text{or} \quad 1\ \forall\, (i, j) \in \bar{A}$$

where $x_{ij} = 1$ if and only if $(i, j) \in A$. X = vector of x_{ij}, $(i, j) \in \overline{A}$; X is a binary representation of a network (N, A). T is computed through an all or nothing user assignment on the shortest paths of the network. Some authors (for instance Los[3]) minimize a sum $T + C$. However the conversion of travel costs into monetary units is rather tricky, so the kind of trade-off represented by the sum $T + P$ is not really known.

Optimal solution

Much literature exists, describing attempts to solve optimally ONP. References can be found in Steenbrink[4] and Los[3]. All authors use implicit enumeration methods which are the only ones capable of coping with combinatorial problems such as ONP. Computational experience has revealed that these methods are practically useless although theoretically successful. This is first due to the complexity of ONP. It belongs to the class of NP-complete problems (see Johnson *et al.*[5]) which cannot be solved optimally by any polynomial growth algorithm. Second, total travel times have to be recomputed each time a link is added or removed. This computation is time-consuming, especially for a removal, since all shortest paths can be unexpectedly modified and have to be re-determined. Practically, optimum-search methods are intractable as soon as the number of nodes n exceeds 10, which of course happens in real cases ($n \geqslant 50$).

Heuristic solution

Approximate methods are the only possible ones when the size of the network is no longer small. A certain number of them can be found in the literature (e.g. Billheimer,[6] Los[3]) but strangely, authors were always interested in minimizing a sum $T + C$, which is a rather disputed criterion index.

Three methods have been carried out for grasping the minimization of total travel time or maximizing accessibility under an investment constraint. They are algorithms of the greedy type; two of them are link removal routines, the other being a link addition one. The main features of greedy algorithms for the case of link removal can be outlined as follows: at each step of the procedure, a search is carried out among the remaining links, to determine the one which, if removed, will provide the best network in the sense of the total travel time. Infeasibility of the solution is then checked; when the solution is feasible, the algorithm stops. If it is an infeasible one, it proceeds to another step.

What differentiates one method from another is the method of evaluation of the link to be removed or added at a given step:

H1 *Method*: a link removal routine in which evaluations $T(A - \{\alpha\})$ of the total travel time in the network (N, A) with link α removed, are computed exactly. This means one has to find shortest paths in every network $(N, A - \{\alpha\})$. In fact bounds, which are formerly computed evaluations are used to limit the number of calls for the shortest path subroutine; this method is in fact a nonoptimal version of that of bounded subsets by Ridley.[7]

H2 *Method*: a link removal routine in which an upperbound $B(\alpha)$ to each $T(A - \{\alpha\})$ is computed avoids the search for new shortest path: for each link α the flow F_α on α is routed on the second shortest path associated with α, whose time is $t_{2\alpha}$, giving:

$$B(\alpha) = T(A) + F_\alpha(t_{2\alpha} - t_\alpha)$$

with

$$t_\alpha = \frac{d_\alpha}{v_\alpha}$$

This bound was first conjectured by Billheimer.[6] However, instead of computing exactly the second shortest path associated with each link, a path made of two joined shortest paths is looked for:

$$t_{2\alpha} = \min_{\substack{k \neq i,j \\ k \in N}} [t_{ik} + t_{kj}]$$

with

α = link ij,
t_{ik} = travel time on shortest path $i - k$.

When the method succeeds, the computing time is very low. If it fails, then:

—either data (travelling times on links) are not homogeneous which is not the case in real systems,
—or the second shortest path is far too long, and link α must be retained, then: $B(\alpha) = \infty$.

H3 *Method*: this is a link addition routine which proceeds in two parts:

—Search for an initial network.

First build heuristically a tree joining every node to every other. Contrary to usual link addition routines in which a minimum spanning tree in the sense of times T or costs C is searched (cf. Steenbrink[4]), consider the trip matrix from the beginning. At each step, a node is determined whose demand for joining the already built part of the tree is maximal. The joining path is then searched which minimizes average total travel time.

—Link addition.

This is performed in a symmetrically identical way as in H1. However, there is no need for recomputing all shortest paths in $(N, A + \{\alpha\})$; only search for a path which would cross link α, if it were added to (N, A), according to Steenbrink.[4] Thus computing time growth is $0\ (n^2)$ instead of $0\ (n^3)$ in H1, n being the number of nodes (see Dubois[8]).

Results

As is shown in Figure 1, H1 is much slower than H2 and H3. Precise computing times are given in Figure 2: problems having between 5 and 53 nodes were first tested for each method, before applying it to the 96 nodes Toulouse network. Every method provides analogous networks in the sense of total travel time, as is shown in Figure 3.

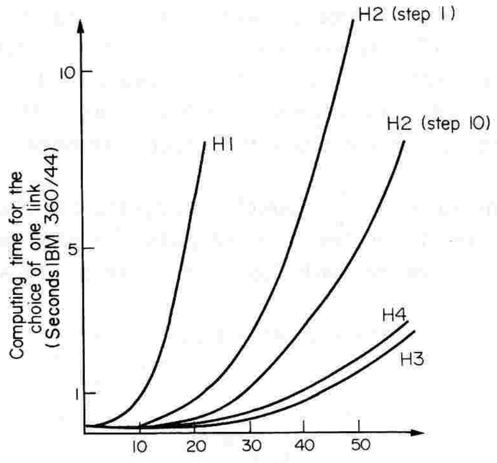

FIG. 1. *Growth of computing time. NB*: H4 is a link addition routine in which links are in the beginning given very high travel times and Part 2 of H3 is at once performed. This figure shows that H3 gives far better results.

Number of zones

n	l	Computing time (seconds IBM 360/44)					
nodes	links	H1	H2*(1)	H2*(5)	H2*(10)	H3	H4
5	10	2	1.5	1.5		1	1
8	18	7	5	5	5	2	2
10	25	24	7	7	7	3	3.5
15	41	112	16	11	10	8	9
30	61		123	62	55	38	46
53	81		512	254	237	125	164
20	61	429	48	20	19	15	22

FIG. 2. *Computing time. NB*: In Figs 1 and 2 "step" is a parameter of H2. It is the number of steps which are performed (= removed links) without updating the shortest paths.

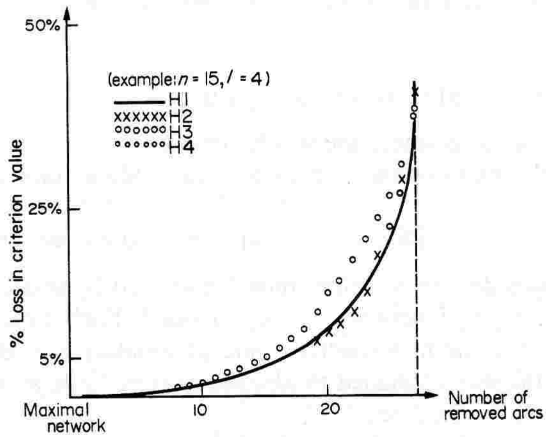

FIG. 3. *Growth of total travel time with the number of chosen arcs.*

FINDING BUS LINES

Given a street network, the question of finding a set of fixed paths for vehicles is scarcely dealt with in the literature (Lampkin and Saalmans[9] a[illegible] Passy[10]).

The first difficulty lies in the lack of objectives for the evaluation of a route system. Intuitively:

—(a) Most trips must, if possible, be direct;
—(b) Lines must not be too numerous;
—(c) Lines must be rather straight and at the same time a sufficient number of passengers must be picked up;
—(d) The number of vehicles must be as low as possible.

The second difficulty is the definition of a candidate set of lines to be added to the network. As in the former approach, frequencies on the routes are not taken into account for the sake of simplicity.

Candidate set

The following is an alternative to the Skeleton method of Lampkin and Saalmans.[10] The basic idea is to use principle (c) to build routes: the first part of that condition prevents the routes from deviating from the shortest paths, second part entails the necessity of crossing a lot of zones. Among the paths which satisfy those demands are "longest shortest paths", i.e. maximal elements of the shortest path set, in the sense of inclusion. Let M be this set. Note that if M were chosen as a set of lines, every trip would be direct. However, M is somewhat insufficient as a candidate set because it may include too short lines which could be joined to others; that is why for every route

$$\lambda = (s_1 \dots s_p) \in M, \text{ the nodes}$$

$$k \neq s_i, i = 1, \quad p$$

are searched for those which have:

$$t_{s_1k} + t_{ks_p} \leqslant t_{s_1s_p}(1 + \eta).$$

I.e. paths between s_1 and s_p, having not too great a length. M^* is thus generated. M can be reduced to $\overline{M}$ by eliminating from M every route which is already included in one of M^*. The candidate set of lines is then $\overline{M} \cup M^*$.

Choice of lines

The method considered involves three parts:

—adding lines so as to connect the whole network;
—searching for the main connection nodes and adding lines which suppress them as possible;
—secondary alteration: to join lines or suppress unused sections.

The first part considers those routes most frequented by users, or the most economical depending on the type of network to be obtained. Each time a route is added, it is necessary to actualize the non-selected route parameters in order to take into account in future choices the users assigned to already existing lines, stopping when the network is totally connected by the lines.

The second part evaluates the number of connections K, the total deviations ΔT and the number of required vehicles ΣB_λ. K is easily obtained when the number U_λ of users on each route are known:

$$K = \sum_{\lambda \in L} U_\lambda - \sum_{i,j \in N} D_{ij}.$$

U_λ depends on the current network which contains λ. ΔT is the sum of the deviations in time δ_λ occurring when using line λ of the current network, instead of the best route in M. Each B_λ is calculated from the number of passengers carried on the most frequented section, assuming the frequencies fitted to the demand. K, ΔT and ΣB_λ globally evaluate the set of lines corresponding to the current network.

To search efficiently for a line which maximizes the decreasing of K, it is possible to evaluate for each zone the number of indirect trips which issue from or end at it. The route to be added is then chosen from those which cross zones having a bad level of service. For each candidate route, evaluate ΔT and the number of connections it would eliminate and its rentability rate; the choice is made on these criteria. The routine is completed when ΔT and K are low enough, provided that the number of required vehicles has not increased too much.

Finally, the improvement routine will consist first in locating, then in suppressing, ill-frequented sections. This suppression will have only limited effect on K and will reduce ΣB_λ. If the cancelled section is not at the route's end, this line will have to be suppressed from the network and a return to phase 2 is sometimes necessary. Furthermore, it is possible to join two adjacent lines, if the result is not too long a path and if it can eliminate connections.

FINDING OPTIMAL FREQUENCIES

Generalities

Subproblem C aims at the definition of optimal frequencies for the lines of the chosen network. The method uses a well known scheme in optimization problems whose structure is non-linear; it needs two steps.

(A) *Evaluation.* For a given set of routes and frequencies, various characterization indices are computed, among which is the total travel time T. The model now takes into account waiting times at bus stops, together with modal choice between private and public transportation, and eventually other kinds of trips.

(B) *Optimization.* According to index values, some new frequencies are computed through an *ad-hoc* gradient-search routine, and step (A) is run again, until convergence of the frequencies occurs.

Planners and decision-makers consider step (A) as very important, because it provides a simulation of the network behaviour, which enables them to forecast frequentation rates.

The evaluation step is described in detail in the following.

EVABUS evaluation model

This model uses the trip matrix D representing the users' demand, assuming that the only alternative to reach a destination is a choice between walking or taking a public bus and the travel times are constant. Hence the time interval between buses on each line, say T_λ, are constant too. The model realizes a thorough search of all possible paths for users; it evaluates waiting and travel times, numbers of passengers for each line, and other indices which may be relevant for a town.

These evaluations are determined using an approach similar to the ones in Lampkin and Saalmans,[9] and Chriqui and Robillard,[11] but the main originality of the model is a new expression of the results, which, in particular, yields directly the gradient of the total travel time with respect to frequencies, whatever the number of lines is, and no differentiation is necessary.

EVABUS includes 3 steps:

(a) Searching for direct trips (without connection) and their characteristics;
(b) Computing the shortest paths in terms of average travel times;
(c) Assigning users to the lines, and computing characteristic indices such as T and its gradient.

Step a. All pairs (i, j) of nodes are checked to locate those which correspond to direct trips using a bus. Direct trips are classified into: bus trips, hybrid trips and walks. The potential frequentation is assumed uniform through the considered period for each line; users' generation times are independent of bus stopping times and lastly the users always choose the shortest path (in time) to reach their destination.

Owing to these assumptions, the waiting time probabilistic distribution function of the number of users at node i is calculated from the knowledge of interval times T_λ for all lines λ which allow a given direct trip. It is then possible to determine the proportion V_{ij}^λ of passengers on each competing line λ realizing this trip, together with the average bus travel time resulting of this distribution function. Hence, the proportion of people making their trip by bus (P_{ij}) or on foot ($1-P_{ij}$) is obtained, so is the average travel time τ_{ij} from node i to node j including walking times, bus travel times, and waiting times. Lastly the gradients:

$$\alpha_{ij} = \left(\frac{\partial \tau_{ij}}{\partial T_\lambda}, \forall \lambda\right)$$

of the average travel times τ_{ij} are calculated using a very fast method (more details can be found in Llibre and Henry[12]).

Step b (shortest paths). From the matrix whose coefficients are τ_{ij}, average travel times for direct trips, those of nondirect trips can be obtained through a shortest path algorithm such as Warshall's.[13] It provides the sequence of connection nodes $k_1, \ldots, k_p$ for any m, n indirect trip such that

$$\tau_{mn} = \tau_{mk_1} + \theta_{k_1} + \ldots + \tau_{k_p n}$$

is a minimum, whatever the number of connections may be, where θ_{k_1} is the connection time at node k_1.

The sequences of connection nodes are stored in a connection matrix $K(m, n)$, which preserves paths.

Step c (Assignment and indices). Bus charges are computed for each line section by section, adding the corresponding terms of the tripmatrix.

For each pair m–n consider the succession of direct trips by using the connection matrix. For a direct trip i–j of m–n pair, the line λ among the competitive lines on i–j will carry the following part of D_{mn}:

$$U^{\lambda}_{mn} = V^{\lambda}_{ij} \cdot P_{ij} \cdot D_{mn}.$$

This charge will have to be added to the one on line λ on all interstations located between i and j nodes.

The same procedure holds for the gradient of T. The corresponding term is:

$$W^{\lambda}_{mn} = \alpha^{\lambda}_{ij}\, D_{mn}.$$

This term is added simultaneously to obtain the following gradient:

$$\frac{\partial T}{\partial T_{\lambda}} = \sum_{m=1}^{N} \sum_{n=1}^{N} W^{\lambda}_{mn}.$$

And the expression for T is:

$$T = \sum_{m=1}^{N} \sum_{n=1}^{N} t_{mn} D_{mn}.$$

Remark

The main original contribution of this network analysis model lies in the use of elementary symmetric polynomial functions for the representation of the waiting time distribution function. This representation easily allows common bus lines to be taken into account. Moreover, the analytical calculation of the average traveltimes gradients is possible.[12] Hence the actual value of the gradient of T computed very rapidly.

Use in optimization routine

The EVABUS evaluation model, giving the criterion gradient directly, is fitted to the search for optimal frequencies. An optimum search heuristic routine has been written and the optimization been used on data obtained from the town of Toulouse considering only the centre town lines and fixing the bus fleet.[12]

According to the starting frequencies, the routine converged towards 2 distinct solutions. Roughly speaking, the total travel time was not altered much by the small frequency changes on the routes when the bus fleet was sufficient and when the frequencies were such that all demands were satisfied.

RUMEUR EVALUATION MODEL[14]

Aim

RUMEUR uses a trip matrix $[D_{ij}]$ with demands from persons travelling inside the town: either with their own vehicles, or walking, or by bus. The aim is therefore to determine the frequentation level of a bus transportation network taking into account the personal vehicles concurrence, the possible connections, the times for reaching stations and waiting times due to bus frequencies.

Data

Since the model is more detailed than the one used in the synthesis, the data, with respect to the various waiting and reaching times, will also be more detailed.

Bus transportation network (TC). This is defined by:

—the graph of bus routes λ_α;
—the rough location of stations which allows determination of the average time to reach station i from zone $i = \theta_{pi}$;
—the bus speed on each route section, which gives the gravel time from station i to station j on route λ: θ^λ_{ij};
—the trip frequencies for each route which gives the average waiting time in station i for the route λ: $\theta^\lambda_{ai} = T_\lambda/2$,
—the possible connections between route λ_α and route λ_β.

Personnel vehicles (PV). A street network is given which the private vehicles use. It is described as usual by a set of road junctions (nodes) and a set of streets (links).

In the modelling, the PV network and the TC one are described in a completely distinct manner, even if buses use the same roads as private vehicles, taking into account the possible presence of reserved paths, the priority given to buses at cross-roads, the stop-times at stations, the eventual presence of a public urban transportation system (subway-PRT...). The global model is therefore a non-planar graph with several dummy nodes and links.

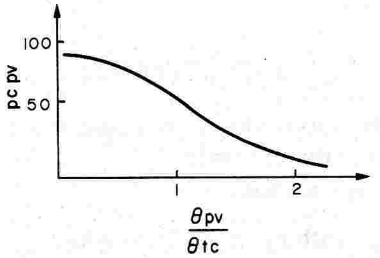

FIG. 4.

From this model yields:
—the travel time from zone i to zone j using a private vehicle θ^{PV}_{ij} to which is later added the time lost for parking θ_{rj},
—the travel time by public transportation θ^{TC}_{ij}.

Modal choice curve. For several types of trips (home to work, leisures, etc....) there are modal choice curves which give for a given ratio θ^{PV} and θ^{TC} travel time, the proportion of people using the PV network $PCPV$ (Figure 4).

Solving method

For each pair of zones (i, j) the shortest paths on the TC network θ^{TC} and on the PV network θ^{PV} are computed.

If, for example, the shortest path TC takes 2 routes λ_1 and λ_2 with connection at station k, then:

$$\theta^{TC} = \theta_{pi} + \theta^{\lambda 2}_{ai} + \theta^{\lambda 1}_{ik} + \theta^{\lambda 2}_{ak} + \theta^{\lambda 2}_{kj} + \theta_{pj}$$

$$\theta^{PV} = \theta^{PV}_{ij} + \theta_{rj}.$$

Using the ratio θ^{PV}/θ^{TC} and to the modal choice curves, determine the portion of users taking the VP, $PCVP$ then the number of users PV:

$$D^{PV}_{ij} = PCPV * D_{ij}$$

and the number of users TC:

$$D^{TC}_{ij} = \frac{100 - PCPV}{100} * D_{ij}.$$

Finally, assign to the various links of the shortest paths *PV* and *TC* the users' flow.

Program outputs

RUMEUR program outputs the following results:

—global TC charge,
—charge per route,
—charge between bus stops,
—the connections used,
—the rough estimate of PV network charge.

Program implementation

The main difficulty is the large size of the graphs required to obtain accurate models of large towns.

It is required to fit a shortest path algorithm in order quickly to solve the problem and minimize computer storage. The program was run on the data gathered from towns having more than 500,000 inhabitants (for instance, for Nice and its suburbs the model had 5400 links, 250 zones and 700 nodes).

APPLICATIONS

Some of the methods described above have been used for the design studies of transportation networks in about ten French towns.

There are two significant applications:

(1) The study for the City Authorities of Nice which, with 500,000 inhabitants, represents the most populated urban center on the coast line of the French Riviera. The RUMEUR model gives the following results:

—loads per section, according to various assumptions;
—necessity of planning a multitype mass transit system;
—required vehicle fleets for each type of public transportation;
—investment cost.

As a consequence of the study, an outstanding effort in favour of the mass transit system has been decided upon by local authorities, and the proportion of investments devoted to this means of transportation will increase, from the VIth Plan (1971–1975) to the VIIth (1976–1980) from 8 to 26%.

(2) The whole method has been applied to a study carried out with the "Centre d'Etude Technique de l'Equipement", "l'Agence d'Urbanisme de l'Agglomération Toulousaine" for the Toulouse bus transportation society, "SEMVAT" (see Bel *et al.*[15]).

The latter wished to determine modifications of their network so as to satisfy the demand in 1985, using eventually the newly developed main roads.

The maximal network included 95 nodes, 64 of them were traffic generating zones, and 164 links.

The H2 method previously described allowed the determination of good subsets of links compatible with a budget constraint. Changing this constraint provided the curve in Figure 5, i.e. the influence of the cost on the total travel time (or any interesting criterion). 20 links were thus suppressed without too much deterioration of the criterion value.

To determine a set of routes which represent the lines, the method described above has been used. A set of 430 routes was then found and evaluated. This candidate set was arranged in increasing order of potential charge and then clustered according to their similarity. Some clusters represented already existing lines.

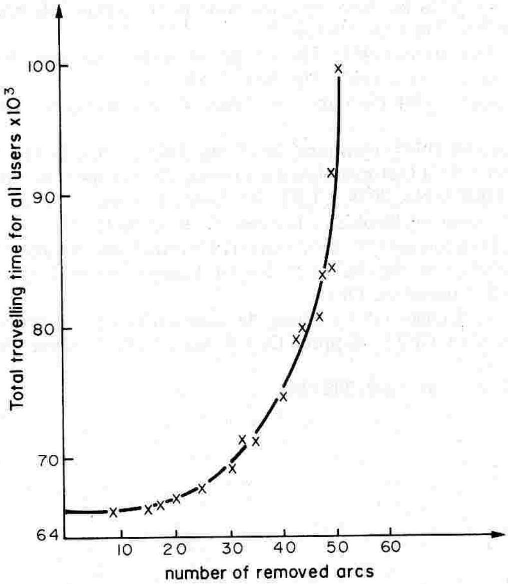

FIG. 5.

A record of the most interesting 50 routes, drawn on the town map was given to the decision maker. Using these proposals, he was able to build 3 networks whose realization could be considered in 1985.

All criteria which must be taken into account in the choice of a network have been evaluated with the EVABUS method described above for the three suggested networks.

In particular, the decision maker received diagrams representing the loads of the various sections and routes.

CONCLUSIONS

These methods show that automatic generation of transportation networks may help a decision maker to find a good solution, i.e. a system which provides a good level of service and moderate investment costs. The appealing feature of those computational methods is their ability to suggest several alternatives and to evaluate them, thus providing some aid to the imagination of planners, as in the Toulouse case study.

Further work can be developed in two directions:

—interaction between the procedure and the decision maker who will guide it towards realistic solutions. This can only be done by breaking up the global choice into partial decisions and by using local criteria.

—introduction of concepts related to the theory of fuzzy sets[16] which will allow an algebraïc processing of ill-known data represented by sets without sharp boundaries rather than single values, in order to deal with the vagueness which pervades transportation planning.

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